Barcelona '10

A031

A 3D Ray-based Pulse Estimation for Seismic Inversion of PSDM Data

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SUMMARY

We develop a methodology for estimation of the point-spread function of prestack depth migrated data (PSDM). The parametrization of the point-spread function is given by a ray-based approach which incorporates the effects of wave propagation through the use of illumination vectors. The only unknown factor in the expression for the point-spread function is a 1D pulse. This pulse is estimated from reflection coefficients in wells, and co-located PSDM data using a least squares approach. The estimate of the point-spread function is the first step towards seismic inversion of PSDM data. In comparison to traditional wavelet estimation, the model using PSDM data has a larger range of validity, i.e. we are able to remove the assumption of a horizontally layered earth. In examples we show that our method is identical to the 1D convolution when the earth has a constant dip, but gives an improvement when this assumption is violated.

Introduction

The 1D convolution model introduced by Robinson (1957) is the industry standard for seismic deconvolution and inversion. This model is based on horizontal layers and approximates post-stack time migrated data. Lecomte et al. (2003) introduced a 3D prestack-depth extension of the convolution approach, which mimics the imaging process of prestack depth migrated data (PSDM). This method acts as a 3D image processing where a reflectivity cube is efficiently filtered by a point-spread function (PSF) to simulate its response through seismic acquisition and PSDM (Lecomte (2008b)). This method still relies on single scattering but includes more of the physics of wave propagation.

Working in a constant-angle domain, our goal is to establish a parallel to well-tie analysis for the PSDM data, i.e. to estimate PSF which correspond to a set of real PSDM data. This is the first step in a 3D inversion. A data-based estimation of PSF requires the full 3D reflectivity to compute the signal, but the observations of reflection coefficients from wells are limited to scattered near-vertical logs. This means that the approach through PSF does not have a direct link between the well-log and synthetic seismic corresponding to the 1D convolutional model. The support of the reflection coefficients in the well is extended by assuming that it is constant along a dipping plane, where the dip angle of the plane vary along the vertical profile.

The PSDM simulator approach of Lecomte et al. (2003) is a ray-based method using simulated point-scatterer PSDM response (also called PSF) at selected image points (Lecomte et al. (2003), Lecomte (2008b), Lecomte (2008a)). Other approaches are also possible (Toxopeus et al. (2008)), but the ray approach provides a flexible, interactive and robust concept for PSF estimation. In the presence of real data, the source pulse is the only unknown parameter in the PSF. This dramatically reduces the dimension of the estimation problem, going from 3D to 1D.

Geophysical model

A key element to calculate PSF with ray tracing is the illumination vectors. For a tutorial description of the relations between these vectors and 3D illumination and resolution effects, see Lecomte (2008b). In brief, the available illumination vectors attached to a survey and calculated in the PSDM background velocity model, will constrain illumination and resolution. The orientation of the illumination vectors indicates the normal to the illuminated reflectors. The shorter this vector is, due to a larger opening angle, the poorer the resolution (pulse stretching effect). The overall spatial coverage of illumination vectors, combined with a pulse, defines the 3D resolution.

Following this model, and in the case of a stationary PSF, f, the seismic PSDM data d for incident angle θ at spatial position (x, y, z) are represented as a 3D-convolution,

$$d(x,y,z;\theta) = \int\int\int c(x-\chi, y-\upsilon, z-\zeta;\theta) f(\chi,\upsilon,\zeta;\theta) d\chi d\upsilon d\zeta + \varepsilon(x,y,z;\theta) \qquad (1)$$

where c are the reflection coefficients and ε is an error term.

Parametrization of the PSF

The PSF at incident angle θ has a wavenumber representation in spherical coordinates relating to the spatial frequencies (k_x, k_y, k_z) by

$$\tilde{f}(r,\phi,\psi;\theta) = \tilde{\alpha}_1(\phi,\psi;\theta)\tilde{\alpha}_2\left(\frac{rV_0}{2S(\theta)},\phi,\psi;\theta\right)\tilde{w}_0\left(\frac{rV_0}{2S(\theta)};\theta\right). \qquad (2)$$

The tilde denotes Fourier transform, w_0 is the source pulse, α_1 and α_2 are known amplitude effects, V_0 is the local velocity and $S(\theta)$ a stretch factor. Amplitude effects which are frequency-independent are modeled trough factor α_1. This includes geometrical spreading, transmission and directivity effects and corrections performed prior to or during migration, e.g. geometrical spreading compensation as a function of traveltime. In an ideal PSDM where all frequency-independent effects are compensated for, α_1 is 1 in the illuminated areas and 0 otherwise. Frequency-dependent amplitude effects such as attenuation and its combined corrections, are modeled trough α_2. In the setting of PSDM data, all parameters in expression (2) are known except from the source pulse. If all amplitude effects were accounted for, we could estimate one common source pulse independent of incident angle θ. In order to account for amplitude effects dependent on incident angle one could also consider to estimate one pulse for each incident angle, in which case we estimate the PSF independently for each angle. For notational simplicity we omit the incident angle, θ, in the remaining part.

Pulse estimation

We assume that seismic data are known at all locations in the region of interest and that well logs give reflection coefficients at well locations. The reflection coefficients are unknown away from the wells. We further assume that the two main axes of continuity at each depth point of the well can be estimated, i.e. to define the plane which is tangent to the layering in each position. This can be done using standard tracking methodology along local extremes of seismic amplitudes. This plane is assumed to be a good approximation to the actual layering in a neighborhood around the well; the dip angles of the plane should change slowly with depth.

Consider the estimation based on one well. For each depth-step ζ along the well the corresponding angles $\phi(\zeta)$ and $\psi(\zeta)$ are the local dip angles. Let c_0 be the reflection coefficient in the well and $k = \frac{1}{S\cos(\psi(\zeta))}$ a dip dependent scale factor. The seismic data in position (x, y, z) in the neighborhood of the well is given by

$$d(x,y,z) = \sum_{\zeta} \frac{Sc_0(\zeta)\alpha_1(\phi(\zeta),\psi(\zeta))}{k(\zeta)} \int w_0(v)\alpha_2(u-v,\phi(\zeta),\psi(\zeta))dv + \varepsilon(x,y,z) \quad (3)$$

where u is a depth-related variable corresponding to a local rotation of the coordinate system by $\phi(\zeta)$ and $\psi(\zeta)$. The integral represents a convolution, and it can be shown that expression (3) can be written on vector-matrix form as

$$\mathbf{d} = \mathbf{G}\mathbf{w}_0 + \boldsymbol{\varepsilon}. \quad (4)$$

This is a multivariate linear regression model, and the least squares estimate for $\mathbf{w}_0$ is

$$\hat{\mathbf{w}}_0 = (\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{d}. \quad (5)$$

In case there are several wells, these data can be stacked together to provide more data. Alternatively one pulse is estimated for each well and the results are pooled afterwards.

A major advantage of the PSF approach is that the estimated source pulse is applicable at all dip angles, while the 1D wavelet estimate assumes constant dip everywhere.

Synthetic 2D case study

The estimation method is illustrated on two synthetic 2D datasets with different PSFs. Filter A in Figure 1 represents a scenario where all dip angles between $-45°$ and $45°$ are equally well illuminated. Filter B represents a scenario with bad illumination for flat or nearly flat and steep ($> 50°$) layers and varying illumination for angles in between. This might indicate a salt body in the overburden. In both cases a Ricker 20Hz source pulse is used.

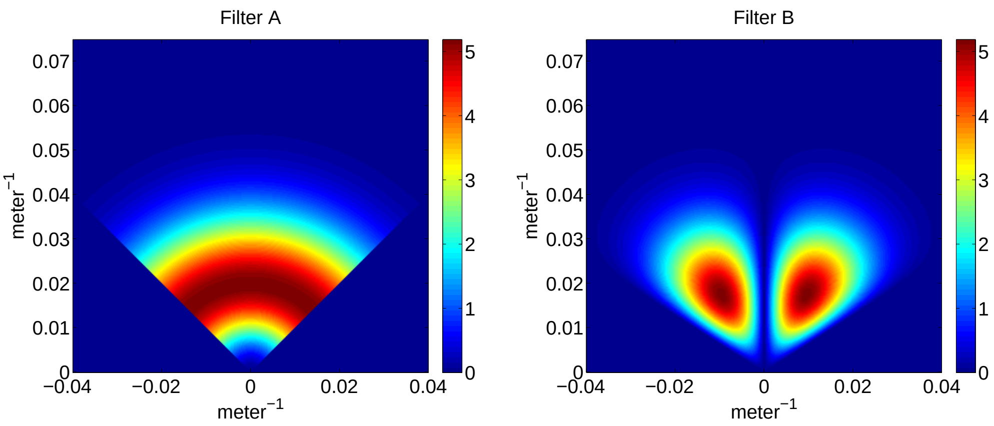

Figure 1 *Left: PSF with constant amplitude scaling for dip angles between $-45°$ and $45°$. Right: Filter with variable scaling for different dip angles.*

Figure 2 (left) shows seismic data generated by convolving filter A with elastic parameters with a sharp reflector with dip 30°. Coloured noise is added giving an overall signal-to-noise ratio of 6. One well at $x = 2000$ is used in the estimation. To illustrate the method, the estimation is based on data from the full depth. In a real case the assumption of stationarity would not apply for such a long trace, but the results would be comparable if multiple wells are used. Figure 2 (right) shows the input pulse (black), the 1D wavelet estimate (blue) and the ray-based pulse estimate (red). The ray-based pulse estimate is very similar to the input pulse, whereas the 1D wavelet estimate is wider. This is due to the stretching of the wavelet and does not indicate a worse fit of the model. Comparing the proportion of the squared error explained by the model with the total squared error (R^2) gives a ratio of 0.85 for both estimates, showing that there is no gain or loss in using the ray-based approach.

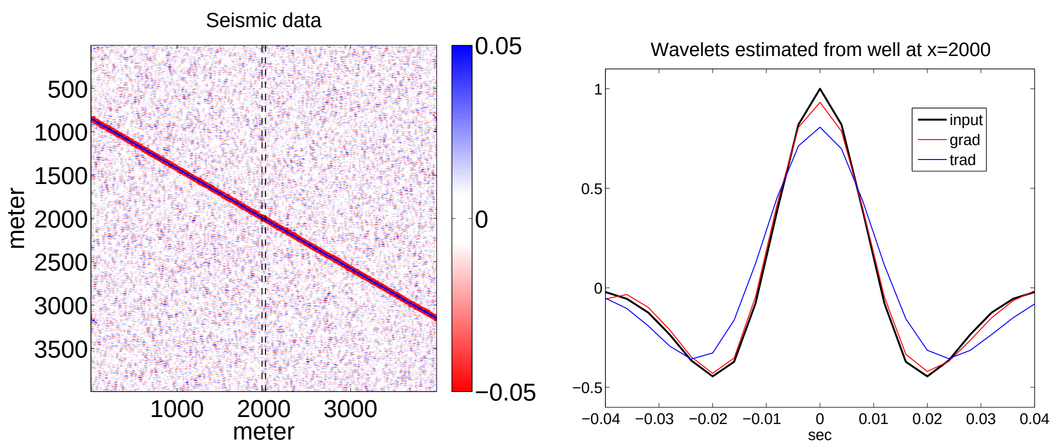

Figure 2 *Left: Seismic data generated by Filter A on an elastic parameter with a sharp reflector with 30° dip. Added coloured noise. Dashed lines indicate well position. Right: Input pulse (black), 1D wavelet estimate (blue), ray-based pulse estimate (red).*

The seismic data to the left in Figure 3 is generated by convolving filter B with elastic parameters divided into three sections. The top and bottom sections have a flat main direction and the middle section has a dip of 30°. The cut between the two lower sections follows the dip angle. Coloured noise is added giving an overall signal-to-noise ratio of 5.3. Two wells are present, at $x = 1000$ and at $x = 2000$. The signals are very weak at the flat areas due to the lack of illumination at angles close to zero, and also at the steepest areas. The well at $x = 2000$ goes through well illuminated areas in all three sections, while the well at $x = 1000$ goes through mainly badly illuminated areas. The local signal-to-noise ratio along the well tracks are 4.5 and 1.7 respectively. This is due to the fact that the noise is stationary and has greater influence in areas with weak signals. The input pulse, 1D wavelet estimates and ray-based pulse estimates are shown in Figure 3. R^2 for the ray-based pulse estimate in the two wells is 0.74 (0.71 for 1D wavelet estimate) and 0.49 (0.22). The well at $x = 2000$ gives a good match because the contribution from the different dip angles are in accordance with the PSF, see expression (2). For the well at $x = 1000$ the long areas with mostly noise gives a poorer estimate.

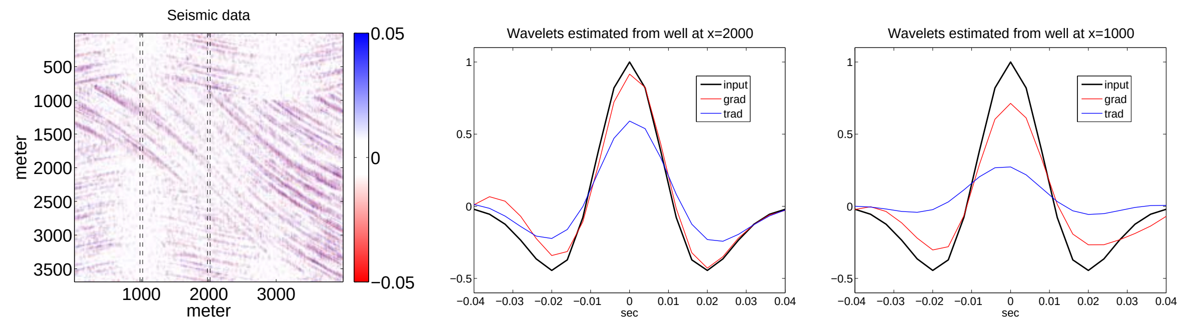

Figure 3 *Left: Seismic data generated by Filter B on elastic parameters from three different sections. The main direction in the top and bottom sections is flat while it has a dip of* 30° *in the middle section. Within each section the layers are curved along the main direction. Middle and right: Estimates from the two wells.*

Conclusions

We have developed a method for estimating the point-spread function matching a set of PSDM data, applicable at all dip angles and tested it in three synthetic examples.

We find that when the assumptions of the 1D convolution model is reasonable, our method provides a fit which is identical to traditional well tie analysis. In cases where the earth is not horizontally layered and contains variable dip, we find a better fit to the data. The gain is obtained by removing a systematic error in the modeling of the seismic amplitudes by including better physical understanding of the wave propagation.

The lack of illumination can however not be compensated by a better model. In regions with a low signal-to-noise ratio, the estimation will still be problematic.

Acknowledgements

The authors thank NORSAR Innovation AS for granting the authorization to publish the present paper. The present work is partially funded by the Research Council of Norway, project #187318/S30. Thanks to all collegaues involved in R&D and software application, both at NR and NORSAR.

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