Gridding for Petroleum Reservoir Simulation

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Abstract

We consider optimization of computational grids for petroleum reservoir flow simulations. In this context grid quality is determined by two independent error sources. On the one hand there is a loss of precision caused by *upscaling* of geological data from the fine geological grid to the coarser computational grid, and on the other hand there are *numerical errors* induced by a non-regular computational grid. In this paper we discuss gridding methods addressing these problems within the restrictions of industry-standard flow simulators.

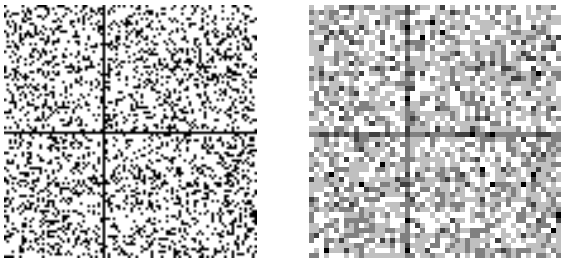
Grid problems in reservoir simulations

The ability to predict the performance of a petroleum reservoir is of immense importance for the petroleum industry. For obvious reasons one would like to be able to know as much as possible about production rates and total production resulting from different production strategies. To this end, numerical reservoir simulation has gained wide acceptance as an important decision-making tool. By reservoir simulation we mean the process of inferring the behaviour of a real reservoir from the performance of a *mathematical model* of that physical system. For our purposes, the model is a set of partial differential equations with an appropriate set of boundary conditions, which describes the significant physical processes taking place in the system. The processes of interest in petroleum reservoirs are basically fluid flow and chemical mass transfer. The model equations must take into account gravitational, capillary, and viscous forces, as well as a reservoir description with respect to permeability heterogeneity and overall geometry.

This paper is concerned with the problem of generating good computational grids for reservoir simulations. Here we face the particular problems connected to heterogeneity and *upscaling*.

The problem of upscaling

One of the inputs to a full field numerical reservoir performance simulator is a reservoir description. This is a model describing a possible three-dimensional map of the geology of the field. Such geological models are often generated by geostatistical methods. The geology is modelled using stochastic simulation conditioned on well observations and other available data. The reservoir description is usually generated on a fine scale, in part reflecting the scale of the input information such as core data. This is done in the belief that the geological model should capture as much as possible of the heterogeneities for accurate predictions of fluid flow. However, to enable a manageable computation, the reservoir performance simulator has to work on a much coarser grid. As a result, one must bring the fine scale permeability data over to a coarser representation. This involves an upscaling or averaging process. This is very problematic, since permeability is a non-additive property.



a Fine grid b Coarse grid

Figure 1: Reservoir divided into four disjunct sections by impermeable barriers.

It is intuitively clear that in problems involving flow in porous media, averaging could lead to severe errors. An example is depicted in Figure 1: Imagine a reservoir divided into four disjunct regions by impermeable walls (Fig. 1a). A simple averaging process into a coarser grid would lead to a smeared-out picture where the no-flow barriers have become low-permeable layers (Fig. 1b). Thus, in this case, the coarse model is qualitatively different from the fine scale model.

The challenge is to upscale with a minimal loss of precision in the predicted reservoir performance. Several ideas have been conceived to solve this problem, ranging from the direct pressure solver method of Warren and Price [1] to renormalization group techniques [2]. Different methods yield different results, and while one method is good for one type of problems another method can be better for another.

However, by recognizing that upscaling involves both the choice of a coarse

grid *and* finding the upscaled values of the geological parametres on this grid, one readily sees that upscaling can be simplified a lot if the coarse grid is as homogeneous as possible before averaging. Needless to say, in the limit of perfect homogeneity, the result of averaging is unique!

Numerical errors

Gridding for petroleum simulators has been relatively conservative with most commercial simulators beeing restricted to structured grids with limited local grid refinement and coarsening. Within this framework we consider two distinct algorithms for grid adaption, namely, grid adaption through

1. node-movement
2. local refinement or coarsening

For both these techniques, grids aligning to the geological structures would generally produce rather distorted grids leading to numerical errors in the equation solvers of the simulator. The complexity of multiphase flow simulation imply that the point at which numerical errors outweighs the advantage of adaption to the geology can only be determined by numerical experiments.

The numerical errors of a solution of a set of differential equations on a grid are caused by the truncation errors due to the discretization. A non-uniform grid produce additional terms in the truncation error. The numerical error and its propagation depend on the differential equations and the discretization method. In elliptic dominated equations like the pressure equation, the local numerical error is closely related to the local truncation error. This is not the case in hyperbolic and parabolic problems, like the saturation equation, where the numerical error propagates easier between regions. Yet, independent of the equation type, it is important to reduce the local truncation error as this is the source of numerical errors. The truncation error is given by higher-order derivatives multiplied by powers of the local grid length. Thus one can reduce the local truncation errors by using a finer grid, but in order to get full effect of a small local grid length, it is necessary to have some distance to larger blocks. Particularly, this is the case if the grid is non-structured with non-neighbour connections [3].

Non-orthogonality will usually imply that cross terms should be added in the finite difference equations. The exact form of these terms can be found by replacing partial derivatives with *covariant* derivatives derived from the metric tensor of the curvilinear coordinate system represented by the non-regular grid. Neglecting these termes induced by a non-trivial metric (this is in fact done in many finite difference simulators) may lead to errors that are independent of the grid spacing. However, even when the metric properties are taken into account, one must be careful. Unless the metric properties are evaluated numerically and with the

same discretization formulae as used in the differential equations, an additional and dominant term occurs in the truncation error (see Ref. [4, p. 62]).

Elastic gridding

Elastic gridding is a variant of variational grid optimization based on a length functional (see e.g., Ref. [5]) with a non-Euclidean metric tensor. In the incarnation of Garcia et al. [6] the grid is considered to be a system of springs connecting neighbouring grid vertices along the grid lines. The problem of grid optimization is thereby reduced to a problem of elasticity and the problem of translating grid optimization criteria into criteria for assigning spring constants to grid lines.

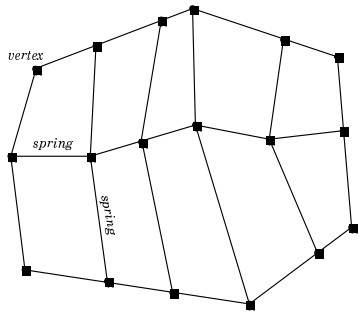


Figure 2: Elastic grid

After having assigned values to the spring constants of the elastic grid, the grid vertex positions can be found by solving the equilibrium equation (Hooke's law) for the elastic system. If we have a system of N vertices, of which M are moveable in one or more directions, then the total elastic energy E of the spring system is

$$E = \frac{1}{2} \sum_{j=1}^{N} \sum_{i \in \mathcal{N}_j} K_{ij} \left[\vec{x}(i) - \vec{x}(j) \right]^2 \tag{1}$$

where $\mathcal{N}_j$ is the set of neighbours to vertex j and K_{ij} is the spring constant of a spring connecting vertex i and j. The vectors $\vec{x}(i)$ and $\vec{x}(j)$ are the position vectors of the two grid vertices.

The corresponding M equilibrium vector equations are

$$\frac{\partial E}{\partial \vec{x}(i)} = \sum_{j \in \mathcal{N}(i)}^{N} K_{ij} (\vec{x}_i - \vec{x}_j) = 0. \tag{2}$$

Thus, we have M vector equations for M vector unknowns. This linear system can be solved at once for all grid vertices.

Since the elastic system is attached to the boundary, it cannot collapse, but for non-convex domains the method meets with the problem of folding. This is the very reason why the length functional is not often considered as a standalone grid generator [5]. We are currently testing ways to solve the grid folding problem by refining the grid or by adding higher-order and non-neighouring couplings. However, for convex domains, the elastic method is very flexible and robust. By letting the spring constants be functions of some local measure of inhomogeneities (as did Garcia et al. [6]), it is possible to make the grid adapt to the geological structures. This can be implemented by letting K_{ij} be a linear function of the variance of sampled values in the grid blocks adjacent to a given spring. In this case $K_{ij} = K_{ij}(\vec{x}_k)$, where k runs over all vertices of the adjacent blocks. Since K_{ij} is determined from sampled values of the neighbouring blocks, Eq. (2) becomes a very complicated non-linear equation. It can be solved iteratively, but to converge one needs to put on some damping mechanism to stop the oscillations that are inherent to any elastic membrane. Two damping techniques have been tried. One is the additive assignment of spring constants proposed by Garcia et al. [6] where $K(n+1) = K(n) + \Delta K$ with the latter term being a function of the variance. In the other method $K(n+1) = 1 + \Delta K$, but the nodes are moved only in the direction of the solution and for each iteration there is a reduction of the step fraction. We have found that the last method gives a somewhat smoother grid.

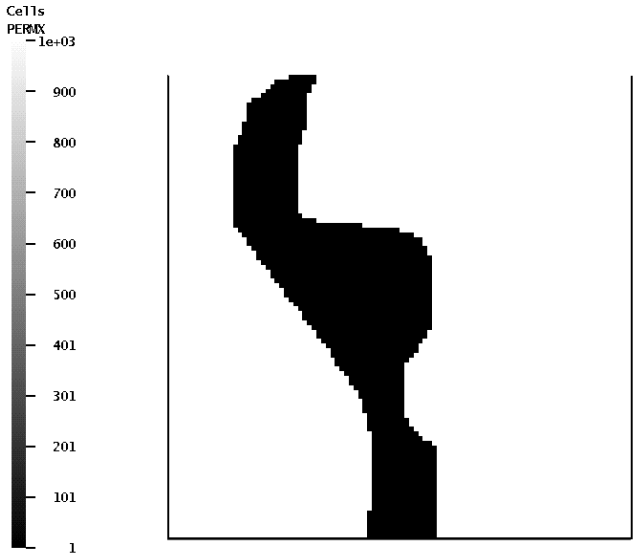


Figure 3: Fine-scale geological model

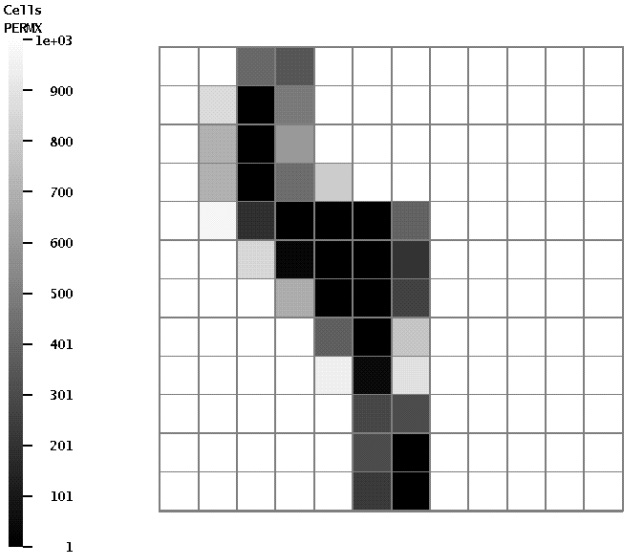


Figure 4: Coarse regular grid

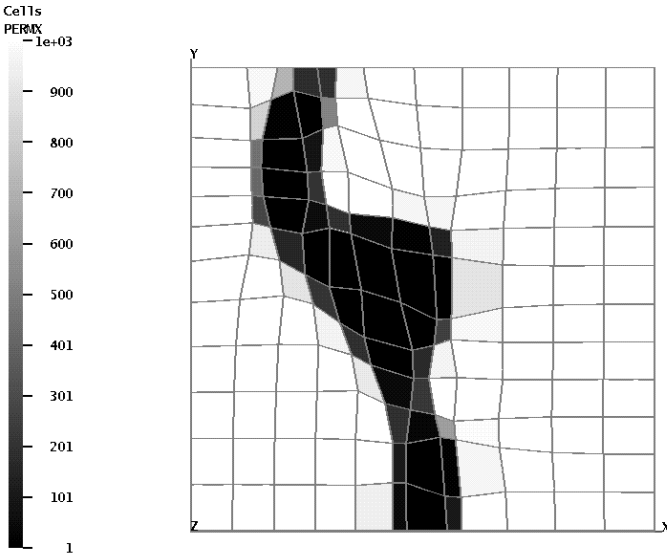


Figure 5: Geology-adapted grid

For simple problems in two dimensions, the grid adapt well to geological structures. Consider for instance, the two-dimensional field in Fig. 3. This model consist of only two materials: one low-permeable medium shown as a black region and a high-permeable medium shown as the white stuff. Upscaling with arithmetic averages leads to the result of Fig. 4. Clearly, a significant blurring has occured. Running an elastic grid optimization program, leads to the grid of Fig. 5. Measured by its ability to represent the fine-scale geological structure (Fig. 3), the grid optimized by means of elastic gridding is a better grid than the regular grid. However, one sees that both orthogonality and grid smoothness has suffered. To a certain degree these measures can be improved by introduing appropriate diagonal couplings (to control deviations from orthogonality) and non-neighbour couplings (to control smoothness).

Dim.	*springs per cell*	*neighb. cells per spring*	*degr. of freedom per cell*
1	1	1	2
2	4	2	8
3	12	4	24

Table 1: Dimension-dependent relations

Unfortunately, in three dimensions with realistic reservoir geological data as input, the method failed. Work is underway to fully understand why this happens. It is clear, however, that the relation between springs and cells is far less direct in three dimensions than in one or two. The geometrical relations for (logically Cartesian) elastic grids listed in Table 1 hints at this fact.

Contrast seeking gridding

Industry standard oil reservoir simulators can handle local refinement or coarsening within a structured grid. One could therefore create a grid that aligns well with geological structures by putting grid cell boundaries at high-contrast surfaces. This idea was put forward by Li, Cullick, and Lake [7]. In their scheme one starts out with a coarse grid. Within each coarse grid cell a moving window searches for high contrasts (the moving window moves with a step length given by a local fine grid), and divides the coarse cell if the variance within the window is larger than a cutoff value. The cutoff variance is adjusted iteratively until a predefined number of grid cells has been obtained. The grid cell sizes are limited below by the size of the fine cells and above by the size of the coarse cells. This guarantees that the sizes of the grid cells do not become too non-uniform.

We have tested a variant of this method in which one starts from a fine grid and a coarse grid. Within each coarse cell, all neighbouring fine cells with per-

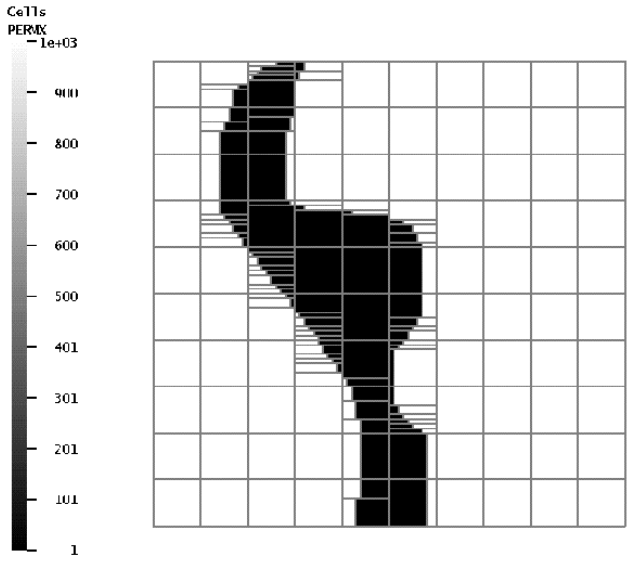


Figure 6: Locally coarsened grid

meability contrasts below a certain limiting value are merged (in order to comply with limitations of the simulators it is checked that the resulting cell is logically Cartesian). For each group of merged cells, the upscaled permeability is computed using simple mean values, but since each coarsened group cover nearly homogeneous regions, the mean value is a good representation. Figure 6 shows the result of starting with a fine grid with the same resolution as the geological grid and a 10 by 10 coarse grid and a limiting contrast of zero. The resulting grid has 229 grid cells and resolves every detail of the geological grid.

This method works as well in three dimensions as in two. Due to the restriction that the coarsened cell groups should be logically Cartesian, there is a tendency for a stronger coarsening in the direction that is coarsened first (in Fig. 6 the grid cells tend to align with the x-direction). This can be utilized to make the grid align to the directions of flowlines. One could therefore imagine that a simulation is carried out on the coarsest grid first, thus establishing the main directions of flow, before choosing the local order of coarsening.

So far our experience is that for petroleum flow simulations, the gain of having a computational grid that is well adapted to the geology outweighs the loss of numerical accuracy by the non-uniformity of the grid.

Conclusion

We have presented and discussed two methods for optimization of petroleum flow simulator grids. Elastic gridding is very simple, and in principle the method should be well suited for weighing between mutually conflicting optimization criteria with highly non-linear and discontinous cost functions. However, for realistic three-dimensional problems, the method does not seem to work. We think that this is due to the fact that grid line springs have a much less direct control over grid cell volumes in three dimensions than in one or two. In addition, the method faces the usual problems of grid folding in non-convex domains. Work is in progress to solve the folding problem by means of higher-order and non-neighbouring couplings in the elasticity equations.

The other method we considered falls within the class of contrast seeking grid optimization. Since the simulator uses a geological model as the arena for fluid flow, we know in advance the exact positions of all permeability contrasts. One could argue that this information should be used directly. By starting with a fine grid and coarsen in homogeneous regions we keep as much of the information about high-contrast surfaces as possible. The restriction of a maximally coarse grid serves as a backbone of structure to avoid the grid to become too non-uniform. The contrast seeking method seems to work very well in all dimensions.

Acknowledgement

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