Upscaling of permeability using global norms

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1 Introduction

We present a new technique for computing the effective permeability in heterogeneous reservoirs on a coarse scale. The method is based on the assumption that the permeability is given at a fine scale and that it is necessary to reduce the number of blocks in the reservoir model.

Traditional upscaling methods depend on local boundary conditions. It is well known that such approaches often lead to non-unique solutions. The new global method is applicable whenever these local methods fail to produce an acceptable permeability field, e.g., for upscaling of blocks close to wells and in cases involving heterogeneities on the coarse grid block scale. In such cases it is impossible to compute an effective permeability based on local observations of the pressure and velocity fields. The properties of the coarse reservoir model simply depend heavily on the global flow pattern.

The basic idea behind the new method is to minimize the global errors introduced in the pressure and velocity fields by the upscaling process. In this approach the total mass flux over each coarse grid block interface will be preserved on the coarse mesh. This leads to very accurate production and injection rates in the wells for the coarse model. Moreover, the associated minimization problem can be solved very efficiently. The solution of the fine scale pressure equation is, however, required. It turns out that, in view of modern numerical methods for elliptic differential equations, the efficiency of the new global scheme is comparable to the performance of the traditional local methods.

We report test results for the new global technique and for the traditional local upscaling method for a fluvial reservoir containing 20 wells. The global approach produces flow patterns that are closer to the fine scale flow field (obtained by simulating directly on the fine scale) than the local methods. The amount of required CPU time is the same for both methods.

2 Global upscaling

Traditional upscaling methods are based on solving the flow equation on each coarse grid-block. Usually a no flow boundary condition is applied to all faces of the block except for two opposite faces where constant Dirichlet boundary conditions are applied. The coarse grid-block permeability is then determined such that the total flow across the block is preserved, see Durlofsky [1] and Warren and Price [4].

In this paper a different approach is studied. As mentioned above, in the global scheme the upscaling problem is formulated as a minimization problem. More precisely, the upscaled permeability is defined as the permeability that minimizes the differences, measured in proper norms, between the pressure and the velocity fields generated by the fine and coarse scale pressure equations, respectively. In the literature addressing parameter identification problems (inverse problems) this approach is often referred to as an output least squares (OLS) method. Therefore we will refer to the new method as the OLS scheme.

Let k_h, p_h and v_h represent the fine scale permeability, pressure and velocity field, respectively. The coarse scale counterparts are represented by k_H (the unknown), $p_H = p_H(k_H)$ and $v_H = v_H(k_H)$ (where we emphasize that both p_H and v_H depend on the unknown coarse scale permeability field k_H). A reasonable ambition on the coarse grid for an accurate upscaling method should be that the pressure in each coarse block Ω_i is close to the average fine scale pressure p_r^i in Ω_i, and that the flux over each coarse grid block interface Γ_i is a good approximation of the average fine flux $q_r^{i,j}$ over Γ_i. It turns out that minimizing the L_2 norms

$$\|p_h - p_H(k_H)\|_{L_2}^2 = \int_\Omega (p_h - p_H)^2 \, dx \tag{1}$$

and

$$\|v_h - v_H(k_H)\|_{L_2}^2 = \int_\Gamma ((v_h - v_H) \cdot n)^2 \, dx \tag{2}$$

give solutions that are close to $p_r = \{p_r^i\}_i$ and $q_r = \{q_r^{i,j}\}_{i,j}$, respectively. Here, Γ represents the union of all the boundaries of the coarse scale grid blocks, n is the outwards directed normal vector to these boundaries, and i, j are block indices (p_r^i is the pressure in block i and $q_r^{i,j}$ is the flux from block i to block j).

In order to minimize the functionals given in (1) and (2), with respect to k_H, it seems to be impossible to avoid computing the fine scale pressure. This leads to a large linear equation system where the number of unknowns is the number of fine scale blocks. However, the most efficient linear equation solvers today use computer time proportional to the number of unknowns, see e.g. [2]. Recall that if local upscaling techniques are applied then a fine scale problem is solved on each coarse block, see e.g. [1]. Thus, the CPU-time needed for solving one fine scale pressure equation, defined on the entire reservoir, is comparable to the time needed by local upscaling methods for solving all of the fine scale problems on the coarse blocks.

In a finite difference reservoir simulator the permeabilities are only used for calculating the transmissibilities. Thus, we will focus on computing transmissibilities, instead of permeabilities, in the algorithm presented below. Clearly, p_r^i and $q_r^{i,j}$ are reproduced on the coarse grid if the transmissibilities satisfy the equation

$$T_H^{i,j}(p_r^i - p_r^j) = q_r^{i,j}, \tag{3}$$

where i and j are neighbor blocks and $T_H^{i,j}$ represents the transmissibility of the associated block interface. This follows easily from conservation of mass and the discretized fine and coarse scale pressure equations, cf. [3].

With this notation at hand our minimization problem can be formulated as follows; Let $T_L^{i,j}$ and $T_U^{i,j}$ be given lower and upper bounds for $T_H^{i,j}$.

L_2 minimization problem

For each pair i, j of neighboring coarse grid blocks find $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ such that $\|v_h - v_H(T_H)\|_{L_2}$ is minimized, and of all solutions that minimize this functional, minimize $\|p_h - p_H(T_H)\|_{L_2}$. In case both functionals are invariant for a transmissibility $T_H^{i,j}$, set $T_H^{i,j} = (T_L^{i,j} T_U^{i,j})^{1/2}$.

In view of the discussion presented above, an approximate solution to this problem can be computed by Algorithm 1.

Algorithm 1

1. Find the solution p_h of the fine scale pressure equation and compute the associated Darcy velocity v_h.

2. Compute the L_2-projections p_r and q_r of p_h and q_h onto the coarse grid.

3. For all pairs of neighbor blocks i, j:

(a) $T_1^{i,j} := q_r^{i,j}/(p_r^i - p_r^j)$,
where $q^{i,j}$ represents the mass flux from block i to block j.

(b) Set $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ such that $|T_H^{i,j} - T_1^{i,j}|$ is minimized.

Hence, in step 3 we try to find $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ satisfying (3). If we can't find any $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ satisfying (3) then $T_H^{i,j}$ is set equal to $T_L^{i,j}$ or $T_U^{i,j}$.

For further details, more accurate algorithms (Algorithm 2 used below) and analyses of their performance we refer to [3].

3 Numerical experiments

We consider a fluvial reservoir with a net/gros equal to 0.4. The reservoir is $11000m$ long, $3000m$ wide, and is having a depth of $50m$. The channel dimensions are given by Gaussian fields, with a wideness with expectation value $500m$ and standard deviation $200m$, and a thickness with expectation value $6m$ and standard deviation $2m$. The reservoir contains 20 vertical wells penetrating the entire reservoir, of which 7 are injection wells and 13 are production wells, see Figure 1. On the fine scale the reservoir is partitioned into $75 \times 75 \times 50$ blocks and the coarse grid is given by $25 \times 25 \times 25$ blocks. Hence the fine scale blocks have the sizes of $147m \times 40m \times 1m$, whereas the coarse scale are $440m \times 120m \times 2m$. The permeabilities in the channels and the background are Gaussian fields with expectation values approximately equal to 54.6 and 1.22, respectively. Similarly, the porosity has an expectation value of 0.2 in the channels and 0.01 in the background.

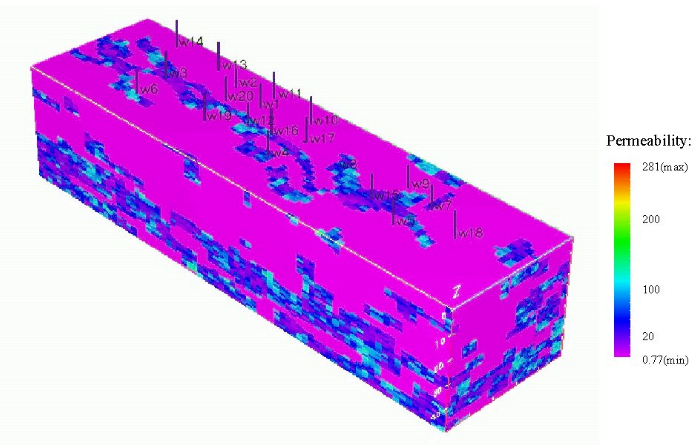

Figure 1: The fluvial reservoir shown with the permeability field and the position of the 20 vertical wells.

3.1 Steady State Solutions

In this part we perform a steady-state test of the reservoir with no flow on the boundaries and fixed pressure in the wells. The specified pressure in the producers and injectors is set to 2 and 3, respectively. We apply two different algorithms for the global upscaling, Algorithm 1 (as described above) and a refined version, Algorithm 2, which both are part of the OLS method and described in detail in the original paper [3]. Essentially, Algorithm 1 finds the transmissibilities of the coarse grid block interfaces by dividing the flow rates between two neighboring blocks by the pressure differences between the blocks. If a transmissibility is outside a permitted interval, the corresponding value is truncated to the value of the closest endpoint for this interval. Algorithm 2 is an iterative extension of Algorithm 1 which modifies the pressures in more and more of the neighboring blocks in order that the ratios between flow rates and pressure differences always are inside the permitted interval for the transmissibilities. The rates in the wells and the pressure and velocity distributions in the entire reservoir are found by solving the fine scale pressure equation. The results of the global upscaling from Algorithms 1 and 2 can be compared to the fine scale solution. The local upscaling is tested by using a Warren and Price type of boundary condition on each coarse grid block.

Algorithm	$\frac{\|p_h - p_H\|_{L^2}}{\|p_h\|_{L^2}}$	$\frac{\|v_h - v_H\|_{L^2}}{\|v_h\|_{L^2}}$
Projection	0.015260	0.67749
Algorithm 1	0.016641	0.68553
Algorithm 2	0.015538	0.69848
Local upscaling	0.021698	0.79674

Table 1: Relative errors in the pressure and velocity fields obtained from the fluvial reservoir.

Well	Fine scale	Algorithm 1	Algorithm 2	Local upscaling
1	827.7	797.6	829.7	474.2
2	614.1	584.3	638.1	277.3
3	685.0	681.3	667.0	413.3
4	1234	1226	1249	775.6
5	1068	1057	1071	684.8
6	386.1	385.9	378.5	275.7
7	1125	1102	1114	702.2
8	−382.6	−384.5	−391.3	−229.0
9	−450.2	−453.3	−440.9	−286.4
10	−170.7	−206.0	−167.9	−122.1
11	−539.1	−421.6	−535.2	−285.2
12	−332.5	−366.5	−327.6	−224.6
13	−330.0	−332.0	−319.5	−226.7
14	−267.7	−270.1	−268.0	−190.7
15	−728.8	−732.5	−749.3	−397.3
16	−428.9	−445.0	−436.4	−312.9
17	−454.5	−462.2	−458.3	−256.2
18	−811.1	−767.4	−798.2	−536.0
19	−414.6	−355.5	−381.7	−197.4
20	−629.7	−636.4	−673.5	−338.4

Table 2: Production and injection rates in the wells obtained for the fluvial reservoir. A calculation of the root mean square of the relative deviations for the rates yields a mean deviation of 3.0% for the rates of Algorithm 2 and a mean deviation of 8.1% for the rates of Algorithm 1. In comparison, the rates due to the local upscaling method are methodically underestimated and deviate on the average by 39.3% from the fine scale rates.

Table 1 contains the relative errors introduced in the pressure and velocity fields by the OLS method (Algorithms 1 and 2) vs. the traditional local upscaling method. The relative errors referred to as “Projection” represent the best approximations on the coarse scale of the fine scale pressure and velocity fields. Hence, it is not possible to obtain smaller relative errors than those given by the projection errors in any type of upscaling. We observe that the relative error of the pressure field for Algorithm 2 is very close to the projection error. For the relative error of the velocity field Algorithm 1 is doing slightly better than Algorithm 2. In both cases, Algorithms 1 and 2 are performing significantly better than the local upscaling method. These observations are confirmed in Table 2. Here the flow rates referred to as “Fine scale” represent the true solution rates. Both Algorithm 1 and Algorithm 2 produce rates that are fairly close to the fine scale rates.

3.2 Perturbations

The OLS scheme is tested by assuming five different sets of boundary conditions[1] for the wells. The different boundary conditions may represent the variation over time for the reservoir. They may also reflect the uncertainty connected to our knowledge of the boundary conditions. Since the OLS method depends on the boundary conditions of the reservoir, the idea here is to test the sensitivity of the global upscaling results against perturbations of these boundary conditions. In these experiments we used Algorithm 2 as defined in [3].

In this perturbation scheme the transmissibilities from the global upscaling are first obtained for each set of perturbed boundary conditions. These transmissibilities are then applied on the coarse grid with the unperturbed boundary conditions. The unperturbed boundary conditions are specified by a pressure equal to 2 in all the production wells and a pressure equal to 3 in all the injection wells. The perturbed boundary conditions are defined by adding a normally distributed variable to the pressures with an expectation value 0 and standard deviations of 0, .1, .25, .5 and 1, for the five sets of boundary conditions respectively. The first case, with a zero standard deviation for this variable, naturally gives the same results as the unperturbed reservoir. A standard deviation of .1 corresponds to 10% of the difference in pressures between the injectors and the producers in the fine scale solution. In this sense the perturbation of the reservoirs from the fine scale solution is 0%, 10%, 25%, 50% and 100%, as indicated in Tables 3 and 4. In our test the actual pressures of the wells with a 10% perturbation were 2.92, 3.08, 2.98, 3.04, 2.84, 3.18, and 2.88 for the injection wells 1 – 7, and 1.99, 1.83, 1.97, 1.99, 2.01, 1.99, 1.92, 1.86, 2.05, 2.16, 2.00, 1.98, and 1.90 for the production wells 8 – 20. We have used the same random numbers for the five sets of boundary conditions such that the perturbation of each well is just a scaling from one set to another. As expected, the results of Tables 3 and 4 show that the results of the OLS method is better the closer the boundary conditions used for finding the transmissibilities are to the actual boundary conditions of the reservoir. A root mean square calculation of the relative deviations for the rates yields deviations of 3.0%, 7.7%, 12.2%, 39.7%, and 30.5% for the 0%, 10%, 25%, 50%, and 100% perturbation, respectively. Given that the rates due to the local upscaling method deviate by 39.3% from the fine scale rates, we conclude that even with a 25% perturbation of the boundary conditions the OLS method gives far better results than the local upscaling technique for this reservoir. In the cases of 50% and 100% perturbation of the boundary conditions, the deviations of the well rates for the OLS method is roughly the same as the corresponding deviation produced by the local upscaling method.

[1] In this paper a set of reservoir boundary conditions is defined as a specification of the pressure or rates in the wells and a specification of the fluxes (in or out) of the reservoir.

Algorithm	$\|p_h - p_H\|_{L^2}/\|p_h\|_{L^2}$	$\|v_h - v_H\|_{L^2}/\|v_h\|_{L^2}$
Projection	0.015260	0.67749
OLS 0%	0.015538	0.69848
OLS 10%	0.015587	0.69718
OLS 25%	0.015893	0.70599
OLS 50%	0.016809	0.76738
OLS 100%	0.019314	0.82625
Local upscaling	0.021698	0.79674

Table 3: Relative errors in the pressure and velocity fields for the perturbed reservoir.

Well	Fine scale	OLS 0%	OLS 10%	OLS 25%	OLS 50%	OLS 100%	Local upscaling
1	827.7	829.7	844.7	878.1	1162	1520	474.2
2	614.1	638.1	615.3	622.1	631.1	582.9	277.3
3	685.0	667.0	710.2	727.4	724.2	692.2	413.3
4	1234	1249	1227	1239	1246	1268	775.6
5	1068	1071	1100	1161	1482	1205	684.8
6	386.1	378.5	370.4	363.0	371.8	390.3	275.7
7	1125	1114	1108	1093	1113	1104	702.2
8	−382.6	−391.3	−440.9	−472.5	−362.8	−448.2	−229.0
9	−450.2	−440.9	−416.5	−379.3	−374.0	−365.9	−286.4
10	−170.7	−167.9	−217.8	−242.2	−441.2	−195.5	−122.1
11	−539.1	−535.2	−526.1	−514.0	−481.5	−1010	−285.2
12	−332.5	−327.6	−327.9	−372.6	−343.7	−428.0	−224.6
13	−330.0	−319.5	−321.5	−319.3	−342.2	−397.8	−226.7
14	−267.7	−268.0	−268.1	−268.1	−258.2	−222.6	−190.7
15	−728.8	−749.3	−714.3	−749.5	−849.8	−911.6	−397.3
16	−428.9	−436.4	−449.4	−435.4	−493.4	−347.4	−312.9
17	−454.5	−458.3	−449.1	−450.9	−515.5	−459.7	−256.2
18	−811.1	−798.2	−806.3	−837.1	−1169	−789.4	−536.0
19	−414.6	−381.7	−392.4	−404.0	−457.0	−427.6	−197.4
20	−629.7	−673.5	−644.3	−638.2	−642.7	−758.9	−338.4

Table 4: Production and injection rates in the wells obtained from the perturbed fluvial reservoir.

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