GLOBAL UPSCALING OF PERMEABILITY.

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SUMMARY

We present a new technique for computing the effective permeability on a coarse scale. It is assumed that the permeability is given at a fine scale and that it is necessary to reduce the number of blocks in the reservoir model.

Traditional upscaling methods depend on local boundary conditions. It is well known that this approach may introduce large errors in the coarse model. The new method is applicable whenever these local methods fail to produce an acceptable permeability field, e.g. upscaling of blocks close to wells and cases involving heterogeneities on the coarse grid block scale. In such cases, it is impossible to compute an effective permeability based on local observations of the pressure and velocity fields. The properties of the coarse reservoir model will simply depend heavily on the global flow pattern.

The basic idea behind the new method is to try to minimise the global errors introduced in the pressure and velocity fields by the upscaling process. This approach to upscaling possesses several desirable properties. More precisely, the total mass flux over each coarse grid block interface is preserved on the coarse mesh. This leads to very accurate production and injection rates in the wells for the coarse model. Moreover, the associated minimisation problem can be solved very efficiently. Traditional, and computational expensive, optimisation algorithms are not needed. However, the solution of the fine scale pressure equation is required. It turns out that the CPU-time needed by the new global scheme is comparable to that needed by the traditional local methods, provided that modern numerical methods for elliptic differential equations are applied.

Finally, the performance of the new scheme is illustrated by a numerical experiment.

1. INTRODUCTION

The last two decades it has become usual to model the permeability in a reservoir at a very fine scale, a scale much finer than it is possible to use in a reservoir simulator. Hence, it is necessary to find effective properties on a coarser scale. However, effective properties are only well-defined if the separation of scale condition is satisfied. This is usually not the case. Traditional upscaling methods compute block effective values of the permeability based on local boundary conditions, cf. e.g. [1] and [2]. We will show that these block effective values depend heavily on the boundary conditions.

In this paper a different approach for upscaling is studied. The upscaling problem is formulated as a minimisation problem. The upscaled permeability is defined as the permeability that minimises the difference between the pressure and the velocity fields generated by the fine and coarse scale pressure equations, respectively. By applying this approach, it is not necessary to

make any assumptions on local boundary conditions. Moreover, we present an effective algorithm for solving the minimisation problem. However, the approach requires the solution of the fine scale pressure equation. These computations are repeated each time the rates or saturations have changed considerably.

The most efficient linear equation solvers today use computer time proportional to the number of unknowns, see e.g. [3]. Thus, the computing time needed for solving one fine scale pressure equation, defined on the entire reservoir, is comparable to the time needed by local upscaling methods for solving all of the fine scale problems on the coarse blocks.

The method calculates the transmissibilities instead of the permeabilities since this is a more stable problem and since it reduces the problem with irregular grid blocks and full permeability tensors.

The method is expected to be applicable for reservoirs where traditional local upscaling techniques fail. This is the case in reservoirs with large contrasts and barriers or high permeable zones at the size of the coarse scale grid blocks. It has also been observed that local upscaling methods quite often fail to handle the well blocks properly.

2. LOCAL UPSCALING METHODS; LACK OF UNIQUENESS

Define the effective permeability for the boundary condition i in direction j to be

$$K_j^i = \frac{< k_h \nabla p_h^i \cdot e_j >}{< \nabla p_h^i \cdot e_j >}, \tag{1}$$

where $< . >$ denotes the average over the block, k_h is the fine scale permeability field, p_h^i denotes the solution of the pressure equation

$$\begin{aligned} &\mathrm{div}(k_h \nabla p_h^i) = 0, \\ &\text{+ boundary conditions,} \end{aligned} \tag{2}$$

in the coarse block corresponding to boundary condition i and e_j denotes the unit vector in direction j. Assume that (2) has been solved for two different Dirichlet boundary conditions $p_h^1 = g^1$ and $p_h^2 = g^2$ on the boundary. Let p_h^3 be the solution of (2) obtained by applying the boundary condition $p_h^3 = a_1 g^1 + a_2 g^2$ for some constants a_1 and a_2. Then also the solution will satisfy $p_h^3 = a_1 p_h^1 + a_2 p_h^2$ inside the coarse grid block. Simple calculation gives

$$K_j^3 = K_j^1 \frac{a_1 < \nabla p_h^1 \cdot e_j >}{a_1 < \nabla p_h^1 \cdot e_j > + a_2 < \nabla p_h^2 \cdot e_j >} + K_j^2 \frac{a_2 < \nabla p_h^2 \cdot e_j >}{a_1 < \nabla p_h^1 \cdot e_j > + a_2 < \nabla p_h^2 \cdot e_j >}.$$

By varying a_1 and a_2, we may get any real value for K_j^3, provided that $K_j^1 \neq K_j^2$. When k_h is not constant in the coarse block, it is possible to find two different values K_j^1 and K_j^2. This shows that the local upscaling problem is not stable. More precisely, for any non-constant fine scale permeability k_h and real number c, it is possible to find a boundary condition where the effective permeability defined by (1) is equal to c.

However, the local effective permeability defined by (1) is stable with respect to small changes in the boundary condition. This is an implication of that the pressure solution and hence also the gradient of the pressure is a continuous function of the boundary condition, cf. e.g. [4].

3. GLOBAL UPSCALING

Let k_h and p_h be as defined in the previous section, except that the fields are defined in the entire reservoir and the boundary conditions are defined by the wells in addition to flux or pressure at the border of the reservoir. The gravity forces are neglected. The associated Darcy velocity is defined by $v_h = -k_h \nabla p_h$.

In this paper we consider the problem of finding a coarse scale description of the permeability k_H such that the corresponding coarse scale pressure p_H and velocity $v_H = -k_H \nabla p_H$ are good approximations of p_h and v_h, respectively. Here, p_H is defined as the solution of the coarse scale pressure equation

$$\operatorname{div}(k_H \nabla p_H) = 0, \tag{3}$$

where we apply the same boundary conditions as in (2). More precisely, for a suitable norm $\|\cdot\|$, we want to find k_H that minimises $\|p_h - p_H(k_H)\|$ and $\|v_h - v_H(k_H)\|$ (we write $p_H(k_H)$ and $v_H(k_H)$ to emphasize that these fields are functions of k_H).

The coarse scale permeability k_H, defined by this approach, will depend heavily on the norm $\|\cdot\|$ we choose. If the energy norm

$$\|p_h - p_H\|_E^2 = \int_\Omega \nabla(p_h - p_H) k_h \nabla(p_h - p_H)\, dx$$

or inverse energy norm (WOLS scheme)

$$\|v_h - v_H\|_{E^{-1}}^2 = \int_\Omega (v_h - v_H) k_h^{-1} (v_h - v_H)\, dx$$

is applied, then it turns out that it is not necessary to solve the fine scale pressure equation (2) in order to compute k_H, cf. [5, 6]. However, as discussed in [6], the energy norm seems to generate results close to the arithmetic mean of the permeability, and the WOLS scheme tend to overestimate the influence of the low permeable zones on the coarse level.

A reasonable ambition on the coarse grid for an accurate upscaling method should be that the pressure in each coarse block Ω_i is close to the average fine scale pressure p_r in Ω_i, and the flux over each coarse grid block interface Γ_i is a good approximation of the average fine flux q_r over Γ_i. It turns out that minimising the L_2 norms

$$\|p_h - p_H(k_H)\|_{L_2}^2 = \int_\Omega (p_h - p_H)^2\, dx \tag{4}$$

and

$$\|v_h - v_H(k_H)\|_{L_2}^2 = \int_\Gamma ((v_h - v_H) \cdot n)^2\, dx \tag{5}$$

give solutions that are close to $p_r = \{p_r^i\}_i$ and $q_r = \{q_r^{i,j}\}_{i,j}$, respectively. Here, Γ represents the union of all the boundaries of the coarse scale grid blocks, n is the outwards directed normal vector to these boundaries, and i, j are block indices (p_r^i is the pressure in block i and $q_r^{i,j}$ is the flux from block i to block j).

In order to minimise the functionals given in (4) and (5), with respect to k_H, it seems to be impossible to avoid computing the fine scale pressure. This leads to a large linear equation system where the number of unknowns is the number of fine scale blocks. However, the most efficient linear equation solvers today use computer time proportional to the number of unknowns, see e.g. [3]. Recall that if local upscaling techniques are applied then a fine scale problem is solved on each coarse block, see e.g. [2]. Thus, the CPU-time needed for solving one fine scale pressure equation, defined on the entire reservoir, is comparable to the time needed by local upscaling methods for solving all of the fine scale problems on the coarse blocks.

In a finite difference reservoir simulator the permeabilities are only used for calculating the transmissibilities. Thus, we will focus on computing transmissibilities instead of permeabilities in the algorithm presented below. Clearly, p_r^i and $q_r^{i,j}$ are reproduced on the coarse grid if the transmissibilities satisfy the equation

$$T_H^{i,j}(p_r^i - p_r^j) = q_r^{i,j}, \tag{6}$$

where i and j are neighbour blocks and $T_H^{i,j}$ represents the transmissibility of the associated block interface. This follows easily from conservation of mass and the discretized fine and coarse scale pressure equations (2) and (3), cf. [6].

With this notation at hand our minimisation problem can be formulated as follows; Let $T_L^{i,j}$ and $T_U^{i,j}$ be given upper and lower bounds for $T_H^{i,j}$.

L_2 minimisation problem

For each pair i, j of neighbouring coarse grid blocks find $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ such that $\|v_h - v_H(T_H)\|_{L_2}$ is minimised, and of all solutions that minimise this functional, minimise $\|p_h - p_H(T_H)\|_{L_2}$. In case both functionals are invariant for a transmissibility $T_H^{i,j}$, set $T_H^{i,j} = (T_L^{i,j} T_U^{i,j})^{1/2}$.

In view of the discussion presented above, an approximate solution to this problem can be computed by algorithm 1.

Algorithm 1

1. Find the solution p_h of the fine scale pressure equation and compute the associated Darcy velocity v_h.
2. Compute the L_2-projections p_r and q_r of p_h and q_h onto the coarse grid.
3. For all pairs of neighbour blocks i, j:
   (a) $T_1^{i,j} := q_r^{i,j}/(p_r^i - p_r^j)$,
   where $q^{i,j}$ represents the mass flux from block i to block j.
   (b) Set $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ such that $|T_H^{i,j} - T_1^{i,j}|$ is minimised.

Hence, in step 3 we try to find $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ satisfying (6). If we can't find any $T_H^{i,j} \in [T_L^{i,j}, T_U^{i,j}]$ satisfying (6) then $T_H^{i,j}$ is set equal to $T_L^{i,j}$ or $T_U^{i,j}$.

For further details, more accurate algorithms and analyses of their performance we refer to [6].

The calculated transmissibilities are continuous with respect to the boundary conditions. However, in the case of large changes in the global boundary condition, it is necessary to recalculate the effective transmissibilities.

It is possible to generalise to multiphase flow by interpreting k_h as the product of the permeability and the relative permeability, see [6]. However, if large changes in the saturations occur it may be necessary to update the effective transmissibilities.

4. A NUMERICAL EXPERIMENT

In this section we will illustrate the performance of the upscaling technique presented above by a numerical experiment. The fine scale permeability data in this example was generated by the program packages FLUVIAL [7] and CONTSIM [8], developed at the Norwegian Computing Center. These commercial software packages, used by several oil companies to model fluvial reservoirs, produce realistic input data suitable for flow simulators.

We consider a fluvial reservoir containing 18 channels, 3 wells (one injector and two producers), and a sand gross of approximately 30%. For simplicity, the pressure in the injector and producers was set equal to 1 and 0.5, respectively.

The permeability data was scaled up from a $20 \times 20 \times 20$ uniform grid to a $10 \times 10 \times 10$ uniform mesh. In addition to the global upscaling algorithm defined above, we applied the classical local method, introduced by Warren and Price in [1], to compute upscaled permeability data on the coarse grid.

In the tables below, the results obtained by our global scheme and the classical method are referred to as *Global upscaling* and *Locale upscaling*, respectively. Next, *Projection* refers to the L_2 projection of the fine scale pressure (or velocity) onto the coarse grid. Recall, that the projection of the pressure/velocity is the best approximation on the coarse scale of the fine scale pressure/ velocity. Hence, the error introduced in the pressure field and the velocity field by any upscaling scheme will always be larger than the projection error. Furthermore, an optimal upscaling technique should generate errors comparable to the projection error. The reader should keep this in mind throughout this section.

Table 1 contains the projection errors and the errors introduced in the pressure and velocity field by the upscaling methods discussed above. Clearly, the global scheme provides almost optimal results, in the sense that the errors are close to the projection errors. We observe that the error in the pressure field is unacceptable for the local method.

Algorithm	$\frac{\|p_h - p_H\|_{L^2}}{\|p_h\|_{L^2}}$	$\frac{\|v_h - v_H\|_{L^2}}{\|v_h\|_{L^2}}$
Projection error	0.0266725	0.650512
Global upscaling	0.0278723	0.733493
Locale upscaling	0.0466163	23.5491

Table 1: Relative error in the pressure and velocity field.

These observations are confirmed by Table 2. Only relatively small errors are introduced in the production and injection rates by **Algorithm 1**, whereas the local method fails to solve this problem.

Similar results have been observed in a series of test problems. Further details and numerical experiments can be found in [6].

Algorithm	well 1	well 2	well 3
Fine scale	2.300	-1.261	-1.040
Global upscaling	2.262	-1.292	-0.970
Locale upscaling	42.9372	-42.1198	-0.817261

Table 2: Production and injection rates in the wells.

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